A METHOD FOR DETERMINING THERMOPHYSICAL CHARACTERISTICS BASED ON THE LAPLACE TRANSFORM

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A method for processing the results of measurements after application of the Laplace transform is proposed for determining thermophysical characteristics by the method of a source of constant power, with the heat capacity of the heater taken into account.

At present a new apparatus for carrying out research has been developed, a systems-structured analysis, which is used effectively for scientific research and solving technical problems.

In [1], the systems-structured approach is applied to an analysis of thermal processes and identification of thermophysical properties of materials and thermal effects. Solution of the problem of heat exchange is given in the form of a block diagram. The elements of the block diagram are the mathematical operators, setting the rules for transforming the thermal and temperature effects influencing an object in the generated reaction. A solution for the problem is sought in the Laplace image space. Thus the mathematical expressions are transformed to expressions that are simpler than the inverse transforms in the time domain.

We consider an application of the systems-structural approach to the problem with a source of constant power. The equation of heat conduction and the boundary conditions, subjected to Laplace transformation with account of zero initial conditions, are of the form [1]

$$a\frac{\partial^2\overline{T}(x,\ p)}{\partial x^2}-p\overline{T}(x,\ p)=0;$$

$$-\lambda\left.\frac{\partial\overline{T}(x,\ p)}{\partial x}\right|_{x=0}=\overline{q}(p)-C_{\text{h}}p\overline{T}(0,\ p);\ \overline{T}(\infty,\ p)=0,$$

where

$$\overline{T}(x,\ p)=\int_0^\infty T(x,\ \tau)\exp(-p\tau)\,d\tau. \tag{1}$$

A solution of this equation, satisfying the boundary conditions, is of the form [1]

$$\overline{T}(x,\ p)=\overline{q}(p)\frac{1}{b\sqrt{p}+C_{\text{h}}p}\exp\left(-\frac{x}{\sqrt{a}}\sqrt{p}\right). \tag{2}$$

For x = 0 we obtain an expression relating the Laplace integrals for the heat flow and for the temperature of a heater

$$\overline{T}(0,\ p)=\overline{q}(p)\frac{1}{b\sqrt{p}+C_{\text{h}}p}. \tag{3}$$

Substituting (3) into (2), we obtain

$$\overline{T}(x,\ p)=\overline{T}(0,\ p)\exp\left(-\frac{x}{\sqrt{a}}\sqrt{p}\right). \tag{4}$$

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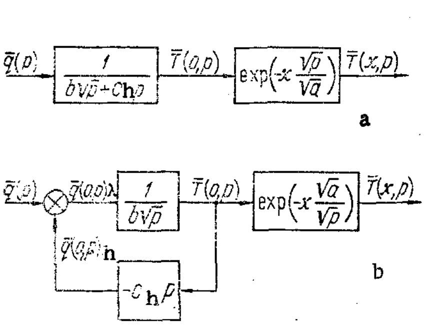

Fig. 1

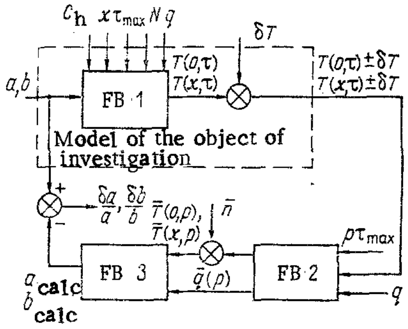

Fig. 2

Fig. 1. Block diagram of a solution to the one-dimensional heat-conduction problem with account of the heat capacity of a heater.

Fig. 2. Block diagram for determining the optimal value of the product $p\tau_{max}$ from the minimum of the relative errors in determining the coefficients of thermal diffusivity and thermal activity: FB1 is the functional block for determining the gradients of the temperatures T(0, τ), T(x, τ); FB2 is the functional block that performs the Laplace transformation; and FB3 is the functional block for determining the value of the thermophysical characteristics.

A block diagram of solutions (3) and (4) is given in Fig. 1a [1]. This block diagram can be represented in an equivalent form, from which it is seen that only a part q_λ of the total heat flow is spent on heating the sample, with another part, q_h, raising the temperature of the heater (Fig. 1b).

From Eqs. (3) and (4), one can obtain formulas for determining the coefficients of thermal activity and thermal diffusivity:

$$b=\frac{\bar{q}(p)}{\bar{T}(0,\ p)}-C_h\sqrt{p}; \tag{5}$$

$$a=\frac{x^2p}{\ln^2\dfrac{\bar{T}(x,\ p)}{\bar{T}(0,\ p)}}. \tag{6}$$

The heat-conduction coefficient is found from the expression $\lambda = b\sqrt{a}$.

The formulas obtained for the coefficients of thermal activity and thermal diffusivity are much simpler than those that would hold for solution of the given probem in the time domain. The integrals of type (1), entering Eqs. (5) and (6), can be easily calculated from the experimental data with the help of Simpson's formula

$$\bar{T}(x,\ p)=\left[\sum_{i=1}^{N-1}T(x,\ \tau_i)\exp(-p\tau_i)\right]\Delta\tau,$$

where $\Delta\tau = \tau_{max}/N$.

For calculating $\bar{T}(0,\ p)$ and $\bar{T}(x,\ p)$ the integral should be taken from zero to infinity, as defined by Eq. (1). However, the function $\exp(-p\tau)\ T(x,\ \tau)$ rapidly approaches zero as τ increases. Therefore, a certain value τ_{max} exists for which the inequality

$$\int_0^\infty \exp(-p\tau)T(x,\ \tau)\,d\tau-\int_0^{\tau_{max}}\exp(-p\tau)T(x,\ \tau)\,d\tau\leqslant e \tag{7}$$

is satisfied where e is a small given number.

Realization of condition (7) depends on the value of the product $p\tau_{max}$; in [2,3] recommendations are given on how to choose it. Thus in [2], it is recommended that $p\tau_{max}$ be selected in the range of 6-30, but in [3] it is recommended that $p\tau_{max}$ be chosen from 8 to 13. Selection of these values is insufficiently accurately justified.

For selection of the optimal value of $p\tau_{max}$ numerical calculations of the errors in determining thermophysical characteristics were conducted, depending on $p\tau_{max}$ and the combinations $u = b\sqrt{\tau_{max}}/C_h$ and $k = x/(2\sqrt{a\tau_{max}})$. The block diagram for determining the optimal value of the product $p\tau_{max}$ is shown in Fig. 2. By specifying the values of the thermophysical characteristics a and b, the heat capacity of the heater C_h, the coordinate x, the maximum duration of the experiment τ_{max}, the number of intervals of N into which the time of the experiment is divided, and the density of heat flow q, we calculated the gradients of the temperature $T(0, \tau)$ and $T(x, \tau)$ for the moments of time $\tau_i = \tau_{max}\ i/N$. The density of heat flow was selected so that the moment of time τ_{max} of the temperature gradient in the zone of the heater $T(0, \tau_{max})$ would be equal to 10°. Such a value of the gradient is usually selected when conducting an experiment in order that the change in thermophysical characteristics of the sample due to heating be negligible, and the accuracy in the measurement of temperature be high enough.

The error in the measurements of nonstationary gradients of temperature δT was simulated with the help of a generator of random numbers, distributed normally with the mean value equal to zero. From the obtained values of the temperature gradients, the values of the Laplace integrals were calculated with the help of Simpson's formula, and the thermophysical characteristics were determined from Eqs.(5) and (6). The obtained values of the thermophysical characteristics were compared with the values a and b, specified when calculating the temperature gradients in the course of solving the direct problem. Then the relative errors in determining the thermophysical characteristics were calculated.

At first we calculated the relative errors in determining thermophysical characteristics due to the finite length of the interval of integration, without including experimental errors. Their values have been found to depend unambiguously on $p\tau_{max}$ and the quantities u and k, where the value of u has a negligible effect on the results of the calculation.

The relative errors in determining the coefficients of the thermal diffusivity and thermal activity will be minimal if in the first case $p\tau_{max}$ is chosen from the relationship $p\tau_{max} = 5\ k + 6$ in the range of values of the quantity k from 0.4 to 1.0, and in the second case $p\tau_{max}$ is equal to 8. The value of u in the calculations was assumed to be 20. The minimal values of these errors constitute not more than 0.2-0.4% and are due to the finite length of the interval of integration and the error in calculating the Laplace integral from Simpson's formula.

When the values $p\tau_{max}$ are less than optimal, the values of the relative errors in determining the thermophysical characteristics increase significantly because of the increase in the contribution of the error due to the finite length of the interval of integration. When the values of $p\tau_{max}$ are higher than optimal, a negligible increase in the errors indicated is observed since only the data for a comparatively short initial interval of time is used for calculating the Laplace integral.

Of greater practical interest is the result obtained by taking account of the error in measuring the temperature gradients, which was defined by changing the dispersion of the generator of random numbers. The values of the relative error in determining thermophysical characteristics was determined for each value of $p\tau_{max}$ and the quantity k for five different combinations of random numbers of the generator. From the results of each of the five calculations, we determined the mean value of the relative error and the root-mean-square deviation. The maximum possible value of the relative error was determined from the expression

$$\delta X/X = |\delta X/X|_m + t\sigma_{\delta X/X}\ , \tag{8}$$

where X is a thermophysical characteristic. The optimal value of $p\tau_{max}$ was determined from the minimum of expression (8).

The dependence of the optimal $p\tau_{max}$ and of the corresponding value of the relative error in determining the coefficient of the thermal diffusivity on the value of the quantity k and on the error in measurement of the temperature is given in Fig. 3. A similar dependence is plotted for the relative error in determining the coefficient of thermal activity (Fig. 4), but this error does not depend on k.

The errors considered are due to the finite length of the interval of integration in computation of the Laplace integral and the error in measuring the temperature gradients. It was assumed that the other values entering Eqs. (5) and (6) were measured without an error. Now we analyze the sources of errors in determining thermophysical characteristics, not treated earlier.

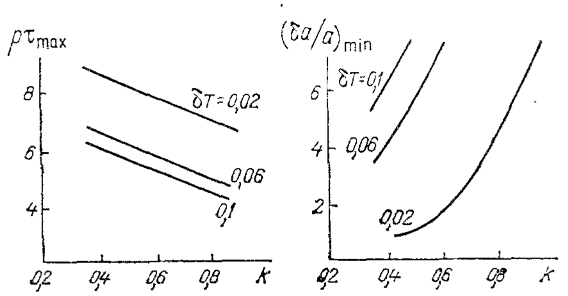

Fig. 3. Dependence of the optimal value of the product $p\tau_{max}$ and of the corresponding value of relative error in determining the coefficient of thermal diffusivity $(\delta \underline{a}/\underline{a})_{min}$ (%) on the quantity k and an error in the measurement of the temperature δT (K).

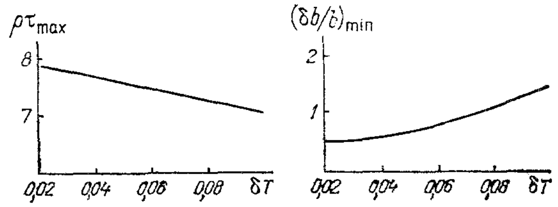

Fig. 4. Dependence of the optimal product $p\tau_{max}$ and of the corresponding value of the relative error in determining the coefficient of thermal activity $(\delta b/b)_{min}$ (%) on the error in the measurement of temperature δT (K).

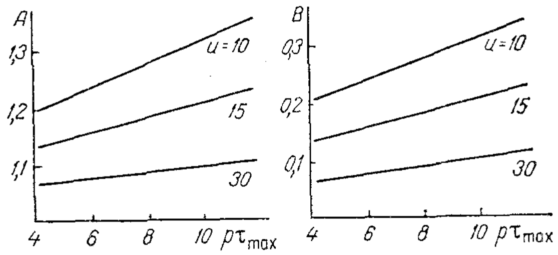

Fig. 5. Dependence of the coefficients A and B on the quantity u and the product $p\tau_{max}$.

The relative error in determining the coefficient of thermal diffusivity due to inaccuracy of measurement of the values entering Eq. (6) is equal to

$$\delta a/a = \sqrt{(\delta a/a_{\Sigma,T})^2 + (2\delta x/x)^2}\,, \tag{9}$$

where $\delta \underline{a}/\underline{a}_{\Sigma,T}$ is the relative error due to the finite length of the interval of integration and the error in measuring the temperature gradients.

The value of x can be measured reasonably simply with an error not exceeding 1%. If in the experiment, the quantity k is not more than 0.4, and the error in measuring the temperature gradients is in the range from 0.05 to 0.10°K, then the relative error will be equal to 4-6%. By decreasing the error in measuring the temperature gradients down to 0.02°K and by decreasing the quantity k (increasing the number Fo_x), the error in determining the coefficient of thermal diffusivity can be reduced to 3%. An increase in the number Fo_x can be achieved either by increasing the duration of the experiment, which requires the use of samples with larger cross-section, or by reducing the size x. A decrease in the size x can lead to an increase in the relative error in determining the thermal diffusivity coefficient according to Eq. (9).

We find the relative error in determining the coefficient of thermal activity due to the inaccuracy in measuring the values entering Eq. (5). We can readily show that this error is equal to the quadratic sum

$$\frac{\delta b}{b}=\sqrt{\left[\frac{(1-e^{-p\tau_{\max}})\,\delta q}{(1-e^{-p\tau_{\max}})q-\overline{T}(0,\ p)\,p^{2}C_{h}}\right]^{2}+\left[\frac{p^{2}\overline{T}(0,\ p)\,\delta C_{h}}{(1-e^{-p\tau_{\max}})\,q-\overline{T}(0,\ p)\,p^{2}C_{h}}\right]^{2}+\left(\frac{\delta b}{b}\right)^{2}_{\Sigma,T}}. \qquad (10)$$

Calculations have shown that the two first terms of the sum (10) depend on the relative errors in determining the heat flow $\delta q/q$ and on the heat capacity of the heater $\delta C_h/C_h$ and also the quantities u and $p\tau_{\max}$.

If $\delta C_h/C_h$ and $\delta b/b_{\Sigma,T}$ are equal to zero, then

$$\delta b/b = A(u,\ p\tau_{\max})\,\delta q/q.$$

A similar dependence exists if $\delta q/q = 0$ and $\delta b/b_{\Sigma,T} = 0$:

$$\delta b/b = B(u,\ p\tau_{\max})\,\delta C_h\ /C_h,$$

where A and B are coefficients depending on the value of the quantity u and $p\tau_{\max}$ (Fig. 5).

Equation (10) can be rewritten in the form

$$\delta b/b = \sqrt{(A\delta q/q)^2+(B\delta C_h\ /C_h\ )^2+(\delta b/b)^2_{\Sigma,T}}. \qquad (11)$$

The value of the relative error in determining the heat capacity of a heater does not have a strong influence on the relative error in determining the coefficient of thermal activity. Thus, for example, for $p\tau_{\max} = 7$, $u = 15$, $\delta T = 0.1$ K, $\delta q/q = 1\%$ and $\delta C_h/C_h = 10\%$, the relative error in determining the coefficient of thermal activity is equal to 2.6%. Calculation without the relative error in determining the heat capacity of a heater yields a result equal to 1.9%. Thus, a contribution to the total relative error from a 10% relative error in determining the heat capacity of a heater constitutes only 0.7%.

It is seen from Fig. 5 that other equal conditions being equal, the smaller value of the contributions to the total relative error of relative errors in determining heat flow and heat capacity of the heater corresponds to the bigger values of the quantity u and smaller values of the product $p\tau_{\max}$. This leads to the fact that the optimal value $p\tau_{\max}$ is somewhat less than the values shown in Fig. 3. The choice of the optimal value $p\tau_{\max}$ should be made for the specific values u, δT, $\delta q/q$, $\delta C_h/C_h$ from the minimum of Eq. (1).

The conducted studies have shown that the method can be effectively used when determining the thermophysical characteristics of the materials.

The dependences obtained in the work can be used to choose the optimal value of the product $p\tau_{\max}$ under different experimental regimes.

NOTATION

x, instantaneous coordinate; τ, time; $T(x, \tau)$, excess temperature with respect to the initial one; a, thermal diffusivity coefficient; b, thermal activity coefficient; C_h, heater heat capacity; q, specific heat flux; p, Laplace transformation parameter; $\delta X/X$, relative error in determining thermophysical characteristics; δT, absolute error in the excess temperature measurement; $\sigma(\delta X/X)$, root-mean-square deviation of the relative error in determining the thermophysical characteristic; t, Student coefficient; Fo_x, Fourier number; u and k, dimensionless quantities; N, number of sampling intervals in the time of the experiment in calculating the Laplace integral from Simpson's formula.

LITERATURE CITED

1. A. G. Shashkov, Systems-Structured Analysis of the Process of Heat Exchange and Its Application [in Russian], Moscow (1983).
2. A. Kavianipour and J. V. Beck, Int. J. Heat Mass Transfer, 21, No. 7, 967-973 (1978).
3. Y. Iida, S. Ohtani, and K. Stephan, "Experimental method to determine the heat production rate, thermal diffusivity, and conductivity of solids," Rev. Sci. Instrum., 21, No. 10, 1648-1653 (1984).